

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 2

Monday 5 June 2023

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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16	
TOTAL	



Answer **all** questions in the spaces provided.

- 1** Given that $y = \sin x + \sinh x$, find $\frac{d^2y}{dx^2} + y$

Circle your answer.

[1 mark]

$2 \sin x$

$-2 \sin x$

$2 \sinh x$

$-2 \sinh x$

- 2** Which one of the expressions below is **not** equal to zero?

Circle your answer.

[1 mark]

$\lim_{x \rightarrow \infty} (x^2 e^{-x})$

$\lim_{x \rightarrow 0} (x^5 \ln x)$

$\lim_{x \rightarrow \infty} \left(\frac{e^x}{x^5} \right)$

$\lim_{x \rightarrow 0} (x^3 e^{-x})$



3

The determinant $A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 2 & 1 \end{vmatrix}$

Which one of the determinants below has a value which is **not** equal to the value of A ?

Tick (✓) **one** box.

[1 mark]

$$\begin{vmatrix} 3 & 1 & 3 \\ 2 & 0 & 2 \\ 3 & 2 & 1 \end{vmatrix} \quad \square$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \end{vmatrix} \quad \square$$

$$\begin{vmatrix} 2 & 2 & 2 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} \quad \square$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{vmatrix} \quad \square$$

4

It is given that $f(x) = \cosh^{-1}(x - 3)$

Which of the sets listed below is the greatest possible domain of the function f ?

Circle your answer.

[1 mark]

$$\{x : x \geq 4\}$$

$$\{x : x \geq 3\}$$

$$\{x : x \geq 1\}$$

$$\{x : x \geq 0\}$$

Turn over ►



Josh and Zoe are solving the following mathematics problem:

Find the equations of the asymptotes of C_2

[5 marks]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 (a) Express $-5 - 5i$ in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$

[2 marks]

6 (b) The point on an Argand diagram that represents $-5 - 5i$ is one of the vertices of an equilateral triangle whose centre is at the origin.

Find the complex numbers represented by the other two vertices of the triangle.

Give your answers in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$

[3 marks]

Turn over ►



7

Show that

$$\sum_{r=11}^{n+1} r^3 = \frac{1}{4}(n^2 + an + b)(n^2 + an + c)$$

where a , b and c are integers to be found.**[3 marks]**



8 **A** is a non-singular 2×2 matrix and $\mathbf{A}^T$ is the transpose of **A**

8 (a) Using the result

$$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$$

show that

$$(\mathbf{A}^{-1})^T = (\mathbf{A}^T)^{-1}$$

[3 marks]

8 (b) It is given that $\mathbf{A} = \begin{bmatrix} 4 & 5 \\ -1 & k \end{bmatrix}$, where k is a real constant.

8 (b) (i) Find $(\mathbf{A}^{-1})^T$, giving your answer in terms of k

[2 marks]

8 (b) (ii) State the restriction on the possible values of k

[1 mark]

Turn over ►



9

$$z = \frac{1 + i}{1 - ki}$$

where k is a real number.

9 (a)

[illegible]

9 (b)

$$\cos \frac{7\pi}{12} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

[5 marks]

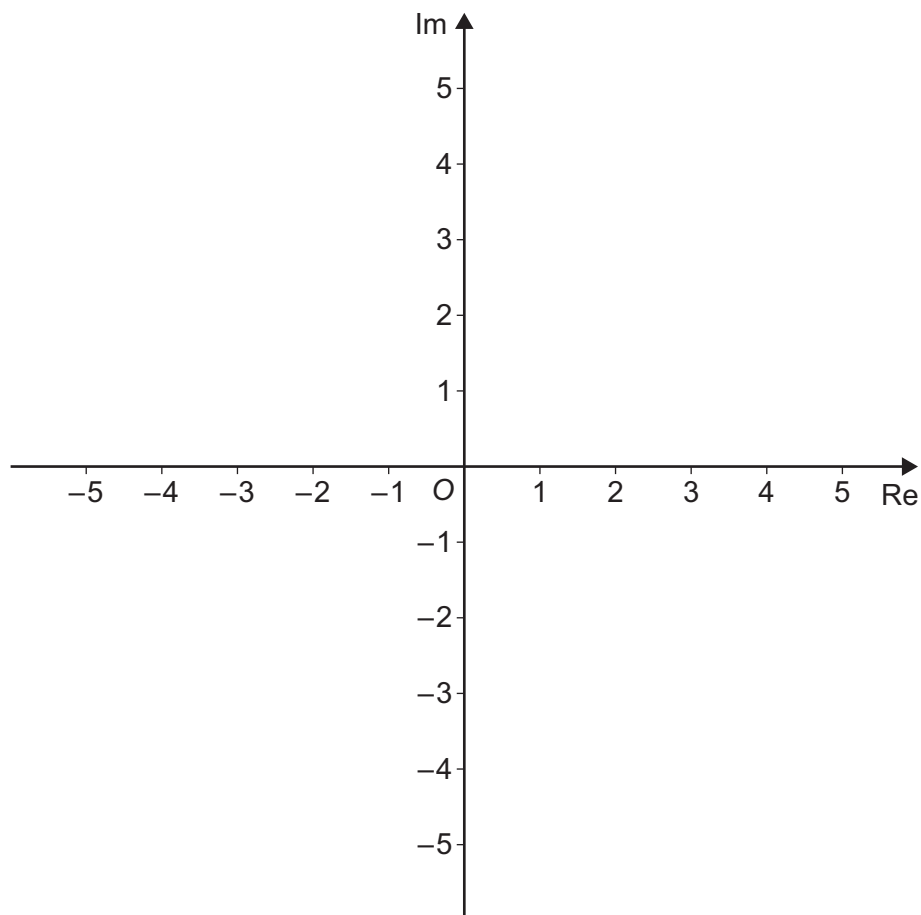


[illegible]

10 The region R on an Argand diagram satisfies both $|z + 2i| \leq 3$ and $-\frac{\pi}{6} \leq \arg(z) \leq \frac{\pi}{2}$

10 (a) Sketch R on the Argand diagram below.

[3 marks]



10 (b) Find the maximum value of $|z|$ in the region R , giving your answer in exact form.

[5 marks]

Turn over for the next question

Turn over ►



11 The line l_1 passes through the points $A(6, 2, 7)$ and $B(4, -3, 7)$

11 (a) Find a Cartesian equation of l_1

[2 marks]

11 (b) The line l_2 has vector equation $\mathbf{r} = \begin{bmatrix} 8 \\ 9 \\ c \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ where c is a constant.

11 (b) (i) Explain how you know that the lines l_1 and l_2 are not perpendicular.

[2 marks]



11 (b) (ii) The lines l_1 and l_2 both lie in the same plane.

Find the value of c

[5 marks]

Turn over for the next question

Turn over ►



The function f is defined by

Prove by induction that $f(n)$ is divisible by 19 for $n \geq 1$

[6 marks]

[illegible]

Do not write
outside the
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[illegible]

Turn over for the next question

Turn over ►



13 The quadratic equation $z^2 - 5z + 8 = 0$ has roots α and β

13 (a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

[2 marks]

13 (b) Without finding the value of α or the value of β , show that $\alpha^4 + \beta^4 = -47$

[4 marks]



[5 marks]

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Turn over ►



14 The function f is defined by

$$f(x) = \frac{1}{4x^2 + 16x + 19} \quad (x \in \mathbb{R})$$

14 (a) Show, without using calculus, that the graph of $y = f(x)$ has a stationary point at $\left(-2, \frac{1}{3}\right)$

[3 marks]

14 (b) Show that $\int_{-2}^{-\frac{1}{2}} f(x) \, dx = \frac{\pi\sqrt{3}}{18}$

[5 marks]



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Fully justify your answer.

[2 marks]

- 15 (a)** Given that $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$z^n - z^{-n} = 2i \sin n\theta$$

[2 marks]

- 15 (b)** The series S is defined as

$$S = \sin \theta + \sin 3\theta + \dots + \sin (2n - 1)\theta$$

Use part (a) to express S in the form

$$S = \frac{1}{2i}(G_1) - \frac{1}{2i}(G_2)$$

where each of G_1 and G_2 is a geometric series.

[3 marks]



$$S = \frac{\sin^2(n\theta)}{\sin \theta}$$
[illegible]

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[illegible]

16 (b) A second, simpler model assumes that the air resistance is zero.

The values of k , m and g remain the same.

Find an expression for x in terms of t according to this simpler model, giving the values of all constants to two significant figures.

[4 marks]

END OF QUESTIONS



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[illegible]